APPLYING STATISTICAL FORECASTING OF PROJECT DURATION TO EARNED SCHEDULE-LONGEST PATH

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ABSTRACT

Project duration forecasting has been enhanced with the introduction and application of the techniques derived from Earned Schedule (ES). The computed forecast results from ES have been shown to be better than any other Earned Value Management based method using both real and simulated performance data. Even so, research has shown that as the topology of the network schedule becomes more parallel, the accuracy of the ES forecast worsens. Recently, forecast accuracy improvement has been achieved for highly parallel type schedules with the method of Earned Schedule-Longest Path. This paper proposes further advancement to the longest path approach through anomaly rejection and the application of statistical methods.

Introduction

To assist the reader's understanding beyond his/her knowledge of Earned Value Management (EVM), three areas will necessarily be discussed: Earned Schedule (ES), Longest Path (LP), and Statistical Forecasting. Some may not need the review. However, for those readers not well-versed in the publications on these topics, the introduction, summarizing several papers, should be helpful [Lipke, 2003; Lipke, 2010; Lipke, 2012-2].

Earned Schedule. The concept is illustrated in figure 1. ES is the measure of time indicating the completed portion of Planned Duration from the EVM performance measurement baseline (PMB). The measure facilitates the ability to assess the schedule performance efficiency; i.e., the time-based schedule performance index, SPI(t). The index is equal to ES divided by AT, the actual time duration from the project start to the status point.
The derived schedule efficiency, SPI(t), in turn, enables the forecasting of project duration through the simple formula [Henderson, 2004]:

IEAC(t) = PD / SPI(t)

where IEAC(t) = Independent Estimate at Completion (time units)
PD = Planned Duration

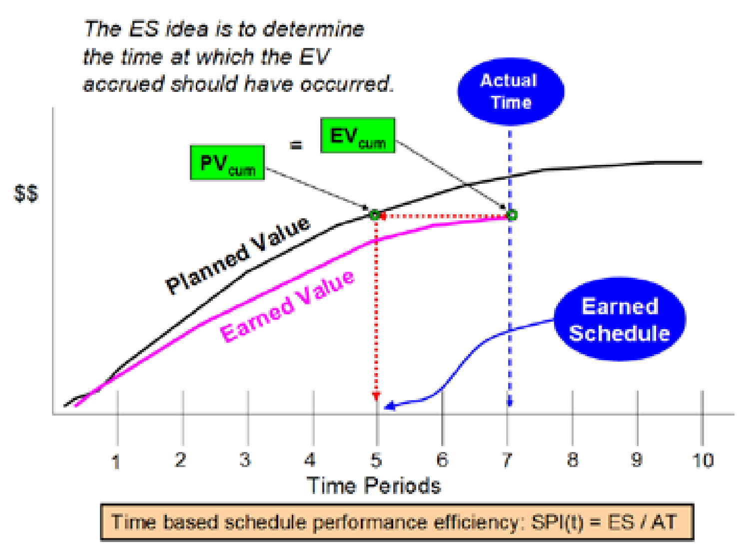

Figure 1. Earned Schedule Concept

The forecasting capability of the formula has been shown to be reasonably good. It has been verified by simulation [Vandevoorde and Vanhoucke, 2007] and application to real project data [Henderson, 2003]. As well, the capability has been verified elsewhere, thereby establishing a compelling argument for applying ES forecasting when EVM is employed for project control [Lipke, 2009; Lipke, 2014]. However, recent research has demonstrated that the topology of the schedule has impact on the "goodness" of the forecast. The ES forecasts are more accurate for schedules which are more serial and less so when parallel [Vanhoucke, 2009].

Longest Path. To improve forecasting for projects having very parallel network schedule topologies, the concept of Longest Path was created. The fundamental idea of LP is that ES forecasting is most accurate from a schedule topology that is completely serial, as shown by Vanhoucke's research cited previously.

Thus, the method requires identifying all of the serial paths in the schedule network leading to project completion. For each path a PMB is created. The longest duration forecast is then determined by applying ES forecasting to each serial path. For the current status point, the longest duration computed from among the paths remaining to be executed is proposed to be the best forecast, and is labeled "LP."

The method has been shown, using notional data, to provide schedule forecasting improvement. Certainly, more research is needed to validate the findings. Nevertheless, at present, the idea appears to provide a solution, albeit complex, to improving ES forecasting for highly parallel project schedules.

Statistical Forecasting. The use of statistical methods for inferring outcomes is a longstanding proven mathematical approach. The statistical forecasting method for duration is relatively simple in concept and, from the statistical hypothesis testing[1] of real data, has been demonstrated to perform rather well [Lipke et al, 2009].

The statistical method of duration forecasting is derived from the ES equation, IEAC(t) = PD / SPI(t), where using the cumulative value of SPI(t) yields the nominal forecast. The associated high and low Confidence Limits[2] are computed from the variation of the periodic values of ln SPI(t):

$$CL = \ln SPI(t)_C \pm Z \times \sigma_M \times AF_S$$

where
CL = Confidence Limit
$\ln SPI(t)_C$ = logarithm of the cumulative value of SPI(t)
Z = the prescribed Confidence Level (usually 90 percent)[3]
$\sigma M = \sigma/\sqrt{n}$, the standard deviation of the sample means
σ = the standard deviation of the logarithm of the periodic values of SPI(t)[4]
n = the number of periodic values
$AF_S = \sqrt{((PD - ES) / (PD - ES/n))}$, the adjustment for finite population[5]

The results obtained from the CL computations are natural logarithms of the cumulative index.[6] In turn, the limit values are used to calculate the estimates of the Confidence Limits for the forecast duration. For example, the high forecast, $IEAC(t)_H$, is calculated using the low CL value, CL(-), as follows:

$$IEAC(t)_H = PD / e^{\wedge}CL(-)$$

where
$CL(-) = \ln SPI(t)_C - Z \times \sigma_M \times AF_S$
e = the base number for natural logarithms

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1. Hypothesis testing is a statistical method for determining the likelihood of the validity of a claim. More information is available from [Wagner, 1992]

2. Information about Confidence Limits may be found in [Crowe et al, 1960]. Confidence Limits are sometimes misunderstood to be thresholds for management action. The limits, instead, describe the region containing the "true" value of the parameter for the prescribed probability, i.e. Confidence Level.

3. Z is a measure of deviation from the mean. Use of the t statistic is recommended when the sample size is less than 30 [Crowe et al, 1960].

4. The logarithm of SPI(t)C is used as the mean value in the computation.

5. The finite population adjustment description may be found in the reference [Lipke, 2009]

6. The use of natural logarithms is attributed to studies of the distribution of the periodic values of CPI and SPI(t) [Lipke, 2002; Lipke, 2012-1].

NOTIONAL DATA / LP ANALYSIS

The notional data used for discussion is shown in table 1. For the example, the project, having 10 tasks, is planned to be completed in 10 periods. The total project and its six paths to completion are depicted with their respective PV and EV values, both periodic and cumulative. The performance path identifiers, such as 1-4-8-10, are the various sequences of the individual task numbers.

To further enhance understanding of table 1 a brief explanation of the symbol “XX” is needed. When used in the PV row, the XX indicates no work was planned for the period, i.e. “Down Time.” In the EV row, the interpretation is the execution was delayed for that period, “Stop Work.” For example, performance was not planned to begin for path 2-4-8-10 until period 3, as shown in the PVp row. For performance path 2-5-9, it is observed that although execution was planned to begin in period 3, it did not commence until period 4. This is shown with XX in the EVp row for periods 1 through 3.

Two paths, 2-5-9 and 6-9 indicate completion two periods past the planned duration of 10 periods. Thus, we know from inspection of the table that execution of the planned critical path (1-4-8-10) did not complete the project and that the longest path must have changed during project execution.

Performance Path	Period	1	2	3	4	5	6	7	8	9	10	11	12
1-4-8-10	PVp	5	5	5	5	5	5	10	5	5	5		
	EVp	XX	4	8	10	3	0	12	8	0	10		
	PVc	5	10	15	20	25	30	40	45	50	55		
	EVc	XX	4	12	22	25	25	37	45	45	55		
2-4-8-10	PVp	XX	XX	10	5	5	5	10	5	5	5		
	EVp	XX	XX	3	11	6	0	12	8	0	10		
	PVc	XX	XX	10	15	20	25	35	40	45	50		
	EVc	XX	XX	3	14	20	20	32	40	40	50		
2-5-9	PVp	XX	XX	10	5	5	5	5	5	5			
	EVp	XX	XX	XX	12	6	5	2	0	4	5	3	3
	PVc	XX	XX	10	15	20	25	30	35	40			
	EVc	XX	XX	XX	12	18	23	25	25	29	34	37	40
3-8-10	PVp	XX	XX	10	10	10	5	10	5	5	5		
	EVp	XX	XX	8	13	9	0	12	8	0	10		
	PVc	XX	XX	10	20	30	35	45	50	55	60		
	EVc	XX	XX	8	21	30	30	42	50	50	60		
7-10	PVp	XX	XX	10	10	10	10	10	XX	5	5		
	EVp	XX	XX	XX	8	9	7	13	8	5	10		
	PVc	XX	XX	10	20	30	40	50	XX	55	60		
	EVc	XX	XX	XX	8	17	24	37	45	50	60		
6-9	PVp	XX	XX	XX	XX	5	5	5	5	5			
	EVp	XX	XX	XX	XX	XX	6	4	0	4	5	3	3
	PVc	XX	XX	XX	XX	5	10	15	20	25			
	EVc	XX	XX	XX	XX	XX	6	10	10	14	19	22	25
Total Project	PVp	5	5	35	30	35	25	25	10	10	5		
	EVp	XX	4	16	43	27	18	31	16	9	15	3	3
	PVc	5	10	45	75	110	135	160	170	180	185		
	EVc	XX	4	20	63	90	108	139	155	164	179	182	185

Table 1. Path Performance

Table 2 contains the computed forecasts for all paths and the total project. For the various paths the longest duration forecast for each status period is identified in the chart by the lime color. Clearly, it is seen that the current LP was identical to the planned CP for only one performance period, period 2. Path 7-10 includes the current LP for periods 4 through 7, while from period 8 through project completion, period 12, longest duration forecasts occur in path 6-9.

Performance Path	**** **** **** Forecast by Period **** **** ****											
	1	2	3	4	5	6	7	8	9	10	11	12
1-4-8-10		13.50	9.33	7.82	9.00	11.00	9.96	9.75	11.00	10.00		
2-4-8-10			28.67	10.89	10.00	12.67	10.51	10.00	11.33	10.00		
2-5-9				8.00	8.38	8.83	10.00	11.75	11.75	11.45	11.75	12.00
3-8-10			12.00	9.62	10.00	12.67	10.51	10.00	11.33	10.00		
7-10				12.75	12.24	12.75	11.57	10.78	11.40	10.00		
6-9						9.17	10.00	12.50	12.14	11.58	11.82	12.00
Total Project		13.50	9.75	9.33	10.03	11.12	10.74	11.29	11.81	11.11	11.64	12.00

Table 2. Longest Path Forecasts

ES for Longest Path. The remaining LP observation, period 3, indicates the longest duration forecast is from path 2-4-8-10 (colored red) and not the lime color cell of path 3-8-10. The value in red is regarded as an anomaly because the representative value of ES for the LP of period 3 decreases from its value at period 2.

For the longest path, the representative ES must increase from one period to the next. This requirement may be understood from the subsequent discussion. However, it raises some fundamental questions. Having a representative value for ES, identified as ES(L), is somewhat confounding. Why is ES(L) different from the ES value used to compute the forecast? To begin the explanation, ES(L) is computed from the path forecast values as follows:

$$ES(L) = PD \times AT / IEAC(t)$$

In contrast, the forecast IEAC(t) is computed from two portions of the path. One is from the executable portion and the other is the number of periods prior to beginning execution. Thus, ES from path execution is different from ES(L); ES represents the executable portion of the path, whereas ES(L) represents the PD of the project.

We now understand that ES(L) is not the same as ES for a path, but why must it increase for successive periods? To respond, let's first assume EV increases from one period to the next. Very simply, from the definition of ES, we can deduce that ES must increase with increasing EV. This analysis applies to both the total project and to the various network paths to project completion.

A more complex situation is EV increases in successive periods for the project, while a path has no accomplishment for the effort made. Referring to table 1, it is seen that this situation occurs for three paths at period 6: 1-4-8-10, 2-4-8-10, and 3-8-10. For this condition, the forecasts will likely increase for both the paths and the project. However, the ES(L) for the paths will decrease. Thus, these paths are excluded from the selection of LP. When EV increases in successive periods for the project, the LP is identified from paths having increasing EV. Because EV increases, ES and ES(L) must increase.

Now, assume for a path that a Stop Work occurs and EV does not increase from the previous period. For this condition ES remains at its value for the previous period. However, even though ES remains the same, the forecast must increase by one period. Because both the forecast and AT increase, ES(L) will increase, as well. This can be deduced by examining the ratio, AT/IEAC(t), from the ES(L) formula. The impact of Stop Work is adding one period to numerator and denominator of the ratio, (AT + 1)/(IEAC(t) + 1). Until the project completes, the value of IEAC(t) will be larger than AT. It then follows, the ratio for the Stop Work period will, likewise, be a larger value than its predecessor; therefore, ES(L) will increase.

From the preceding discussion the following has been established:

Due to the requirement for ES(L) to increase, the selection of LP for a period is conditional. LP is chosen as the longest forecast having a positive change in ES(L).

The discussion to this point covers the possibilities of project performance, with one exception. Disregarding Stop Work, it is possible, although highly improbable, for a project to have no EV for a performance period. For this situation, the LP is indeterminate and the period is excluded from forecasting.

Performance Path	**** **** **** ES(L) byPeriod **** **** ****											
	1	2	3	4	5	6	7	8	9	10	11	12
1-4-8-10		1.48	3.21	5.12	5.56	5.45	7.03	8.21	8.18	10.00		
2-4-8-10			1.05	3.67	5.00	4.74	6.66	8.00	7.94	10.00		
2-5-9				5.00	5.96	6.79	7.00	6.81	7.66	8.73	9.36	10.00
3-8-10			2.50	4.16	5.00	4.74	6.66	8.00	7.94	10.00		
7-10				3.14	4.09	4.71	6.05	7.42	7.89	10.00		
6-9						6.55	7.00	6.40	7.41	8.64	9.31	10.00
Total Project		1.48	3.08	4.29	4.98	5.40	6.52	7.08	7.62	9.00	9.45	10.00

Table 3. ES(L) Values

ES(L)-LP Forecasting. From the following brief discussion, the reader should realize a clear understanding as to how LP was selected for period 3. Using the formula for ES(L), table 2 is transformed into table 3. The ES(L) values of table 3 provide the means to identify the positive change needed for LP selection. From the table, it is observed that the ES(L) value for period 3 of path 2-4-8-10 is less than the value for period 2 of path 1-4-8-10 (1.05 < 1.48), thus identifying the anomalous forecast. The next lowest value of ES(L) at period 3 is then chosen to satisfy the increasing requirement. The ES(L) value of 2.50 identifies path 3-8-10, thereby selecting 12.00 as the LP forecast for period 3, indicated by the lime color in table 2.

The result of applying the ES(L) condition, LPc, is illustrated by figure 2. The plot of LP forecasts without the condition is yellow, while for LPc the graph is orange. Both LP and LPc provide improved forecasts in comparison to the total project; the two overlay for periods 4 through 12. Additionally, LPc provides an improved forecast for period 3, highlighting the anomalous forecast of LP.

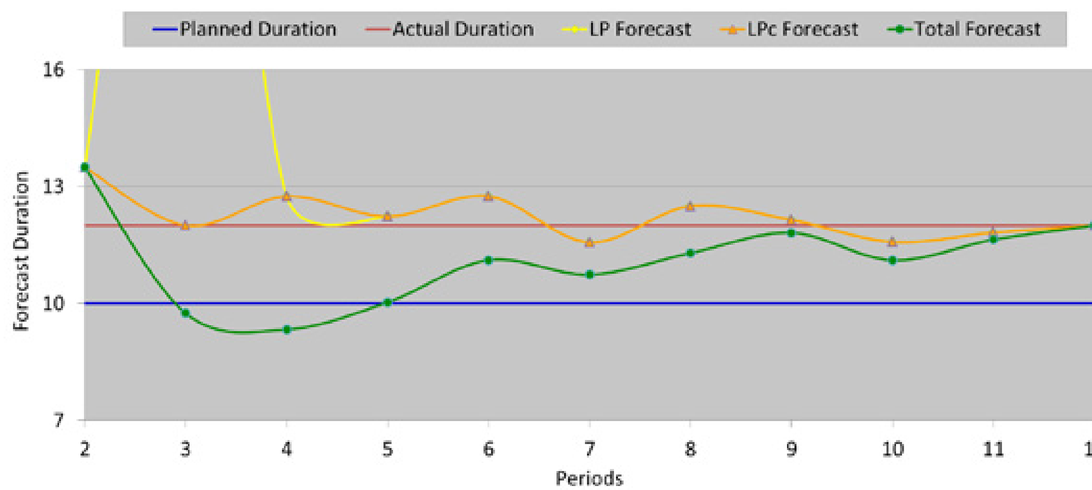

Figure 2. Comparison of Duration Forecasts

ES(L)-LP STATISTICAL FORECASTING

Initially it was thought that the application of statistics to ES longest path might not be possible. Creating the capability to compute Confidence Limits (CLs) seemed out of reach. At best, it was envisioned that applying statistics to LP would be overly burdensome and highly complex.

Nevertheless, with the promise that LP holds for improving forecasting for highly parallel schedule networks, the effort to develop the statistical application was believed warranted. Expecting the worst, it was a pleasant surprise to find the solution is amazingly simple, as the reader will discover.

Upon inspection of the requirements, it was determined that only one item is needed. From it everything else is derived. The lone requirement is the ability to compute periodic values for SPI(t). These values, in turn, are used to compute the standard deviation, the critical component of the CL calculations.

To obtain periodic SPI(t), all that is needed are periodic values of ES, regardless of their attribution. Thus, using ES values from the total project will yield its set of statistical forecasts. And, correspondingly, the ES(L) values provide its corresponding forecasts. Simple enough.

The results of the statistical forecasting using ES(L)-LP values from the notional data are shown in figure 3.[7] The graphs associated with ES(L)-LP are annotated by appending their identifying label with "c." Also shown are the CL plots from the ES-LP values. Larger variation is observed due to the anomaly in period 3; the CLs for ES-LP forecasts are further from the nominal forecast. Also noted for both sets of CLs, the graphs are reasonably symmetrical about the nominal forecast. Although symmetry of CLs isn't a requirement or even an expectation, it does provide a sense that the forecasting method is providing reliable management information.

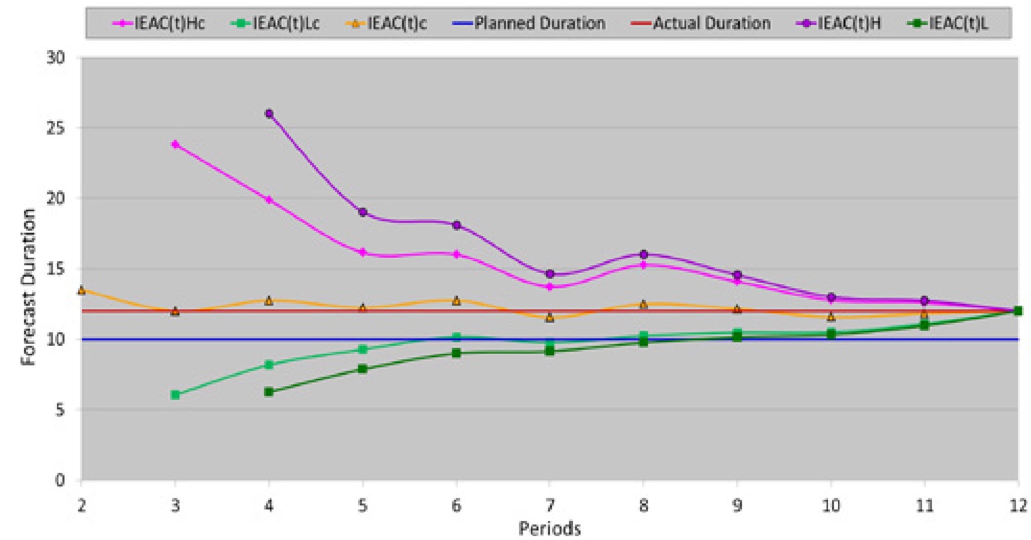

Figure 3. ES(L)-LP Statistical Forecast

Another observation from figure 3 to be noted is period 3 CL values are absent for the ES-LP graphs. This is due to the anomaly identified for period 3. The negative change in periodic ES, discussed previously, causes problems with the calculations involving logarithms; for real number results, the logarithm function excludes negative argument values. Thus, because of the negative change in ES(L) between periods 2 and 3 discussed earlier, we have the reason for the period 3 omission; it is not calculable.

The statistical forecast from ES for the total project (TP) is provided in figure 4. Figures 3 and 4 taken together facilitate comparison between the LP methods and the total project. As observed, all forecasts converge to the actual duration of 12 periods; this point is important and oftentimes is overlooked [Lipke, 2014]. It is noted that the difference between the high and low CL is slightly greater for the LP methods for the first few periods; this is believed reasonable because the size of the data sample for the TP is much greater.

Although the TP variation is smaller overall, the LP methods, especially ES(L)-LP, are considered to be an improvement. The symmetry of CLs around the nominal forecast and, more importantly, the forecast accuracy throughout the project duration indicate the ES(L)-LP forecasting method may justify the additional analysis effort.

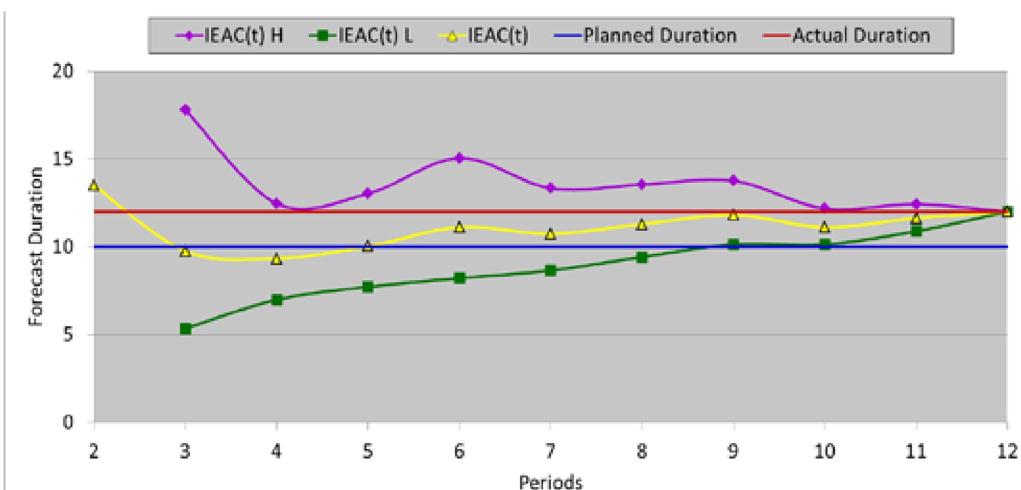

Figure 4. Total Project Statistical Forecast

7. Confidence Level of 90 percent was used for the calculations.

Of course, if ES(L)-LP proves to be an improvement through additional research and application, and subsequently automated tools are created for the statistical forecasting, then the drawback of added effort is largely removed. For researchers and early adopters, some assist is offered by three Excel files available for download from the ES website (www.earnedschedule.com): ES Calculator vs1c, ES-LP Calculator v1b, and Statistical Forecasting Calculator v2c.

The vs1c and v1b calculators are used to compute path forecasts and ES(L) values. When Stop Work or Down Time conditions are encountered the vs1c calculator is needed, otherwise v1b is recommended due to its relative simplicity. Having the ES(L) values for the LP selections, the v2c calculator is then utilized to obtain the statistical forecasts.

SUMMARY AND CONCLUSION

Using EVM data, the forecasting of project duration with ES has proven, through research and application, to be very reliable. Its accuracy, however, has been shown in recent research to decrease as the topology of the schedule network becomes increasingly parallel. To counter this deficiency and improve ES forecasting the concept of LP has been proposed. From trials with notional data, LP appears to improve forecasting for highly parallel schedules.

For identifying the LP from all of the serial path forecasts, it was determined that a condition is required to preclude anomalous selection. The LP selected for a performance period is the longest duration forecast having a positive change in the representative value of ES for the path, ES(L).

Initially believed to be difficult and complex, statistical forecasting was determined to be reasonably straightforward. It is accomplished by using the ES(L) values associated with the LP selections identified at the periodic status points of project performance. From the ES(L) values the standard deviation is derived, which then allows the computation of the CLs.

As depicted in figures 3 and 4, the application of ES(L)-LP is an improvement to ES-LP and TP statistical forecasting. Although the sample is small, it is believed this finding is generally true.

In conclusion, research is recommended to validate/reject the described methods and the claimed project duration forecasting improvement. If validated, then it becomes reasonable to automate the methods, thereby increasing the practicality of their application. Although it will take some time, the vision appears achievable.

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